# Short Report: Factorizable Geometric Reasoning in Hierarchical Associative Memory

## 1. Overview

Factorizable geometric reasoning refers to the ability of a memory or reasoning system to decompose complex spatial or relational structures into reusable, independent factors (e.g. position, orientation, scale, part–whole relations) and recombine them efficiently. In hierarchical associative memory (HAM), this capability emerges when representations are structured so that different geometric or relational dimensions are encoded in separable subspaces and bound together through controlled associations.

This approach contrasts with monolithic distributed representations, where geometric relationships are entangled and must be relearned for each new configuration.

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## 2. Hierarchical Associative Memory (HAM)

Hierarchical associative memory organizes knowledge across multiple abstraction levels:

- **Lower levels** encode local, concrete features (edges, points, primitive relations).
- **Intermediate levels** encode parts, motifs, or invariant local configurations.
- **Higher levels** encode global structure, schemas, or relational graphs.

Associations are typically learned via co-activation, correlation, or predictive coding, allowing partial input to retrieve complete patterns. Hierarchy enables reuse: the same lower-level patterns participate in many higher-level constructs.

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## 3. Factorization of Geometric Structure

Factorization in geometric reasoning means representing structure as a product of independent components:

- **Shape × Pose** (what vs. where)
- **Object × Transformation**

- **Topology × Metric detail**
- **Part identity × Part arrangement**

In HAM, factorization is achieved by:

- Assigning separate representational channels or subspaces to each factor.
- Using binding operations (e.g. tensor products, circular convolution, key–value associations) to compose factors.
- Allowing each factor to be recalled or modified independently.

This enables systematic generalization: a learned shape can be applied to a new pose without relearning the entire configuration.

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## 4. Geometric Reasoning via Association

Geometric reasoning emerges as structured memory retrieval:

- **Completion**: partial spatial input retrieves missing components.
- **Analogy**: shared factors allow mapping between different scenes or objects.
- **Transformation inference**: changes in one factor (e.g. rotation) are detected while others remain stable.

Hierarchical levels support reasoning at different geometric scales: local metric adjustments at lower levels, global relational reasoning at higher levels.

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## 5. Advantages of Factorizable HAM

- **Compositionality**: new structures formed by recombining known factors.
- **Data efficiency**: fewer examples needed due to reuse of components.
- **Robustness**: noise in one factor does not corrupt the entire representation.

- **Interpretability**: factors correspond to meaningful geometric variables.

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### 6. Relation to Existing Frameworks

Factorizable geometric reasoning in HAM connects to:

- **Vector Symbolic Architectures (VSA)** and **Holographic Reduced Representations**
- **Capsule networks** (explicit pose–part separation)
- **Predictive coding hierarchies**
- **Graph neural memories with factorized node/edge embeddings**

These systems differ in implementation but share the principle of separable structure bound through association.

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### 7. Open Challenges

- Learning clean factorization without supervision.
- Preventing factor leakage and entanglement at higher levels.
- Scaling associative retrieval to large, continuous geometric spaces.
- Integrating symbolic constraints with continuous geometry.

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## 8. Reinterpreting Deep ReLU Networks as Hierarchical Associative Memory

An alternative and unifying perspective is to view a plain deep ReLU-based neural network itself as a form of hierarchical associative memory. Under this interpretation, each layer implements a learned associative map between partially active input patterns and increasingly abstract internal representations.

ReLU activations partition the input space into a large number of linear regions, each corresponding to a distinct pattern of neuron activation. Within any such region, the network behaves as a linear associative system, mapping input vectors to output vectors through fixed affine transformations. Across layers, these regions are composed hierarchically, yielding a multi-level memory structure that associates low-level geometric features with higher-level relational constructs.

From this viewpoint:

- **Association** arises through learned weight matrices that bind co-occurring features.
- **Hierarchy** emerges from layer-wise composition, progressively increasing receptive field size and abstraction.
- **Memory retrieval** corresponds to forward inference, where partial or noisy inputs activate nearby regions that reconstruct stored patterns.
- **Generalization** reflects shared activation regions encoding common geometric or relational factors.

Although standard deep ReLU networks do not explicitly enforce factorization, factor-like structure can emerge implicitly through distributed representations. Different subspaces may become sensitive to shape, pose, or relational variables when this improves predictive performance, effectively approximating the behavior of a factorizable hierarchical associative memory.

This reinterpretation suggests that many geometric reasoning capabilities attributed to specialized memory architectures may already exist in conventional deep networks, but in an entangled and implicit form. Making this structure explicit—through architectural constraints, regularization, or auxiliary objectives—may be viewed not as introducing a new paradigm, but as refining and stabilizing a latent associative memory already present in deep ReLU systems.

## 9. Conclusion

Factorizable geometric reasoning in hierarchical associative memory provides a powerful mechanism for structured generalization and efficient spatial reasoning. By separating *what* from *how* and *where*, and organizing these factors across hierarchical levels, such systems can achieve flexible, interpretable, and data-efficient reasoning that bridges symbolic and sub-symbolic computation.